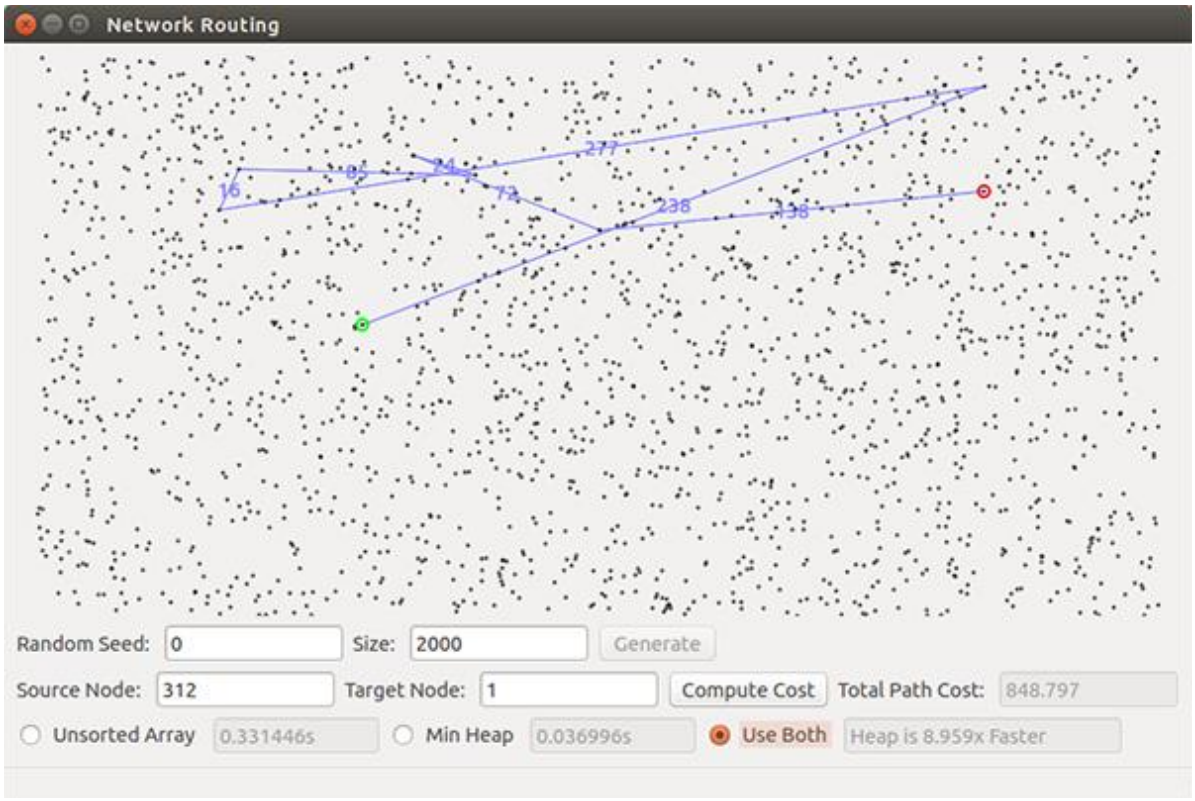
Project #3: Network Routing (Dijkstra's Algorithm)



Framework: The framework provides:

1. A graphical user interface that generates a graph/network with a specified number of points, for a provided random seed
2. A display for the graph vertices and for the subsequently computed shortest paths

When you hit "Generate" the framework for this project generates a random set of nodes, *V*, each with 3 randomly selected edges to other nodes. The edges are directed and the edge cost is the Euclidean distance between the nodes. Thus, all nodes will have an out-degree of 3, but no predictable value for in-degree. You will be passed a graph structure which is comprised of $|V|$ nodes, each with 3 out edges, thus $|E| = 3*|V| = O(|V|)$. The nodes have an (x,y) location and the edges include the start/end nodes and the edge length. The nodes are drawn on the display in the provided framework. You can hit "Generate" again to build a new graph. Each random seed leads to a different random graph.

After generating, clicking on a node (or entering its index in the appropriate box) will highlight it in green as the source node, and clicking another node (or, again entering its index in the box) will highlight that node in red as the destination. Each click alternates between the two. After these nodes are selected you can hit "Compute", and your code should draw the shortest path starting from the source node and following all intermediate nodes until the destination node. Next to each edge between two nodes, display the cost of that segment of the route. Also, in the "Total Path Cost" box, display the total path cost. If the destination cannot be reached from the source then put "unreachable" in the total cost box. Clicking on the screen again will clear the current path while allowing you to set another source/destination pair.

The "Compute" button should call two different versions of Dijkstra's, one that uses an *array* to implement the priority queue, and one that uses a *heap*. (Use the standard Dijkstra's algorithm from the text which puts all nodes initially in the queue and finds the shortest path from the source node to all nodes in the network. After running that, then you just need to display the path from the source to the destination.) Both priority queue versions should give you the same path and cost, but will have different efficiencies. Each time you hit solve

and show a path, display the time required for each version, and the times speedup of the heap implementation over the array implementation.

To make sure everything is working correctly, first try some smaller problems that you can solve by hand. Also, at the top of this page, we have included a screen shot for a particular 2000-vertex problem, which you can use as a debug check to help ensure your code is working correctly. At the bottom there is an additional 1000-vertex problem you can use for comparison and debugging.

Note that for the examples created by our interface, since each node has a predefined maximum out-degree, $O(|E|) = O(|V|)$. This will affect your empirical analysis. However, your algorithm handles all cases, including those where $|E|$ is much larger than $|V|$. Thus, your complexity analysis (#3 below) should use the general case for $|E|$ and $|V|$.

In a stable network, a node could just run Dijkstra's once for every node in the network and set up a routing table which it could then use for any message. However, for unstable networks (new nodes, outages, etc.) such a table would not necessarily be correct for very long, and you might run Dijkstra's each time like we are doing here.

Instructions:

1. Implement Dijkstra's algorithm to find shortest paths in a graph.
2. Implement two versions of a priority queue class, one using an unsorted array (a python list) as the data structure and one using a binary heap.
    - For the array implementation, *insert* and *decrease-key* are simple O(1) operations, but *delete-min* will unavoidably be $O(|V|)$.
    - For the binary heap implementation, all three operations (*insert*, *delete-min*, and *decrease-key*) must be worst case $O(\log|V|)$. For your heap implementation, you may implement the heap with an array, but remember that *decrease-key* will be $O(|V|)$ unless you have a separate array (or map) of object references into your heap, so that you can have fast access to an arbitrary node. Thus, you *must* use the separate lookup map. Also, don't forget that you will need to adjust this lookup array/map of references every time you swap elements in the heap.

You may NOT use an existing heap implementation. You must implement **both** versions of the priority queue from scratch.

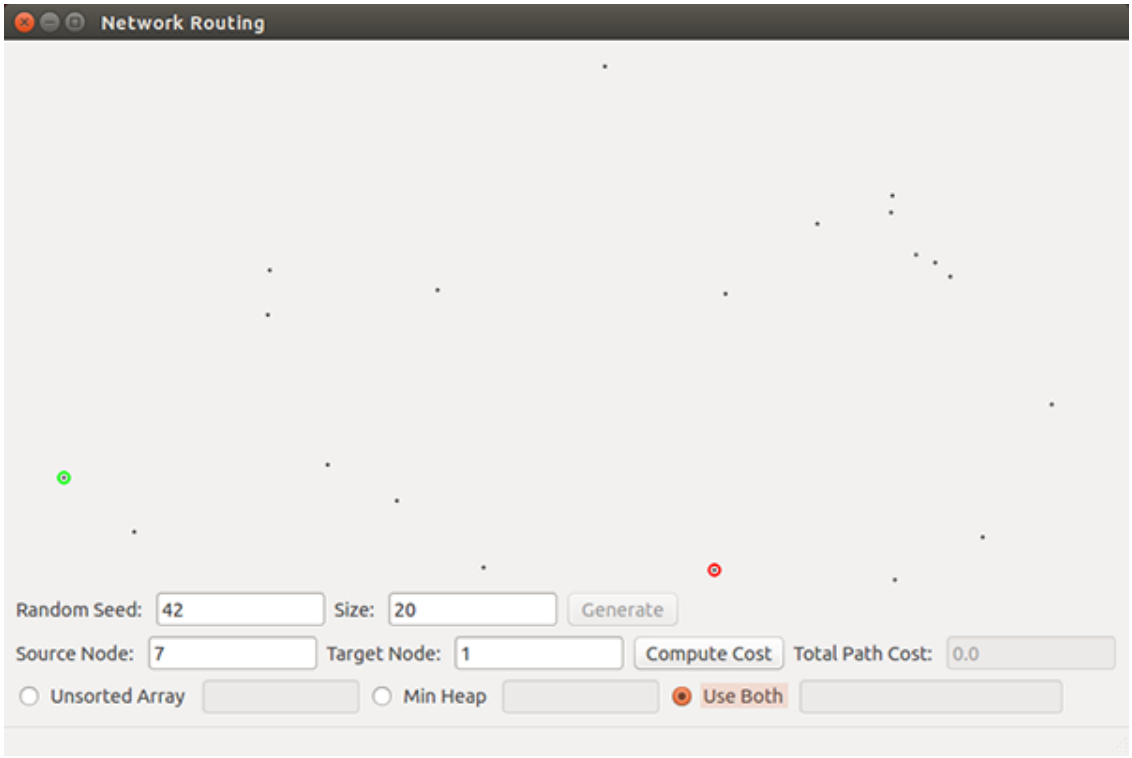
Report: 90 points total. The other 10 come from your design experience.

1. [15] Correctly implement Dijkstra's algorithm and the functionality discussed above. Include a copy of your documented code in your submission.
2. [15] Correctly implement both versions of a priority queue, one using an array with worst case O(1), O(1) and $O(|V|)$ operations and one using a binary heap with worst case $O(\log|V|)$ operations.
3. [20 points] Discuss the time and space complexity of the overall Dijkstra algorithm and each of your two versions with your priority queue implementations. You must demonstrate that you really understand the complexity and which parts of your program lead to that complexity. You may do this by:
    a. Showing and summing up the complexity of each significant subsection of your code, or
    b. Creating brief psuedocode showing the critical complexity portions, or
    c. Using another approach of your choice.

    For whichever approach you choose, include sufficient discussion/explanation to demonstrate your understanding of the complexity of the entire problem and any significant subparts.
4. [20] For Random seed 42 - Size 20, Random Seed 123 - Size 200 and Random Seed 312 - Size 500, submit a screenshot showing the shortest path (if one exists) for each of the three source-destination pairs, as shown in the images below.

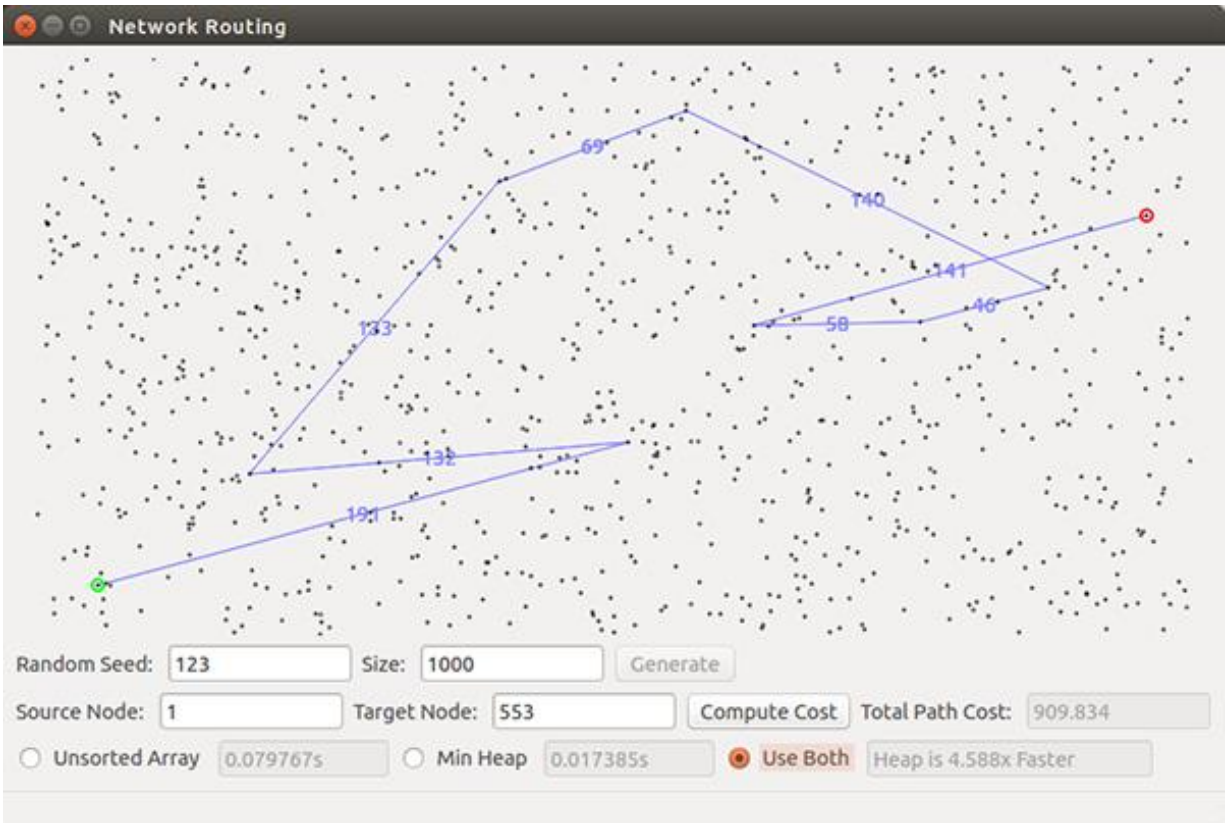
a. For Random seed 42 - Size 20, use node 7 (the left-most node) as the source and node 1 (on the bottom toward the right) as the destination, as in the first image below.
b. For Random seed 123 - Size 200, use node 94 (near the upper left) as the source and node 3 (near the lower right) as the destination, as in the second image below.
c. For Random seed 312 - Size 500, use node 2 (near the lower left) as the source and node 8 (near the upper right) as the destination, as in the third image below.

5. [20] For different numbers of nodes (100, 1000, 10000, 100000, 1000000), compare the empirical time complexity for Array vs. Heap, and give your best estimate of the difference. As a sanity check, typical runtimes for 100,000 nodes is a few mintues for the array and a few seconds for the heap. For 1,000,000 nodes, run only the heap version and then estimate how long you might expect your array version to run based on your other results. For each number of nodes do at least 5 tests with different random seeds and average the results. Graph your results and also give a table of your raw data (data for each of the runs); in both graph and table, include your one estimated runtime (array implementation for 1,000,000 points). Discuss the results and give your best explanations of why they turned out as they did.




Network Routing
Random Seed: 123
Size: 200
Generate
Source Node: 94
Target Node: 3
Compute Cost
Total Path Cost: 0.0
Unsorted Array
Min Heap
Use Both


Network Routing
Random Seed: 312
Size: 500
Generate
Source Node: 2
Target Node: 8
Compute Cost
Total Path Cost: 0.0
Unsorted Array
Min Heap
Use Both


Network Routing
69
140
141
46
58
133
132
191
Random Seed:
123
Size:
1000
Generate
Source Node:
1
Target Node:
553
Compute Cost
Total Path Cost:
909.834
Unsorted Array
0.079767s
Min Heap
0.017385s
Use Both
Heap is 4.588x Faster