Quantum associative memory

One of the most promising approaches to quantum neurocomputing is the quantum associative memory, of which one approach is described in [33-35]. The task of pattern association can be broken down into two major components: memorization and recall. The memorization step consists of storing patterns in the memory while the recall step entails pattern completion or pattern association based on partial and/or noisy input.

Memorization

An efficient quantum algorithm for constructing a coherent state over n qubits to represent a set of m patterns is presented in [29]. The algorithm is implemented using a polynomial number (in the length and number of patterns) of elementary operations on one, two, or three qubits. The key operator in this process is

$$\hat{S}^p = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \sqrt{\frac{p-1}{p}} & \frac{-1}{\sqrt{p}} \\ 0 & 0 & \frac{1}{\sqrt{p}} & \sqrt{\frac{p-1}{p}} \end{bmatrix}$$

where $m \geq p \geq 1$. This is actually a set of operators that are conditional transforms – there is a different $\hat{S}^p$ operator associated with each pattern to be stored. The algorithm also makes use of various versions of some standard quantum computational operators such as the Controlled-Not and Fredkin gates. Now given a set P of m binary patterns of length n, the quantum algorithm for storing the patterns requires a set of $2n+1$ qubits, the first n of which actually store the patterns and can be thought of as n neurons in a quantum associative memory. The remaining $n+1$ qubits are ancillary qubits used for bookkeeping and are restored to the state $|\bar{0}\rangle$ after every storage iteration. Each iteration through the algorithm makes use of a different $\hat{S}^p$ operator and results in another pattern being incorporated into the quantum system. The result is a coherent superposition of states that correspond to the patterns, with the amplitudes of the states in the superposition all being equal. The algorithm requires O(mn) steps to encode the m patterns as a quantum superposition over n quantum neurons. This is optimal in the sense that just reading each instance once cannot be done any faster than O(mn).

Recall – completion

The recall capability of the quantum associative memory can be implemented using the quantum search algorithm due to Grover [36]. This algorithm has been traditionally considered as implementing a search for an item in an unsorted (quantum) database of N items, and it performs this task in O($\sqrt{N}$) time, a feat that is impossible classically. In the quantum computational setting, finding an item in the database means measuring the system and having the system collapse to the basis state which corresponds to the item in the database for which we are searching. Now, we can equally well consider the algorithm as accomplishing the task of pattern completion in a quantum associative memory. The basic idea of Grover's algorithm is to invert the phase of the desired basis state and then to invert all the basis states about the average amplitude of all the states. Repetition of this process produces an increase in the amplitude of the desired basis state to near unity followed by a corresponding decrease in the amplitude of the desired state back to its original magnitude. The process has a period of $\frac{\pi}{4}\sqrt{N}$ and thus after O($\sqrt{N}$) operations, the system may be observed in the desired state with near certainty. Define

$$\hat{I}_{\phi} = \text{identity matrix except for } \; i_{\phi\phi} = -1$$

which inverts the phase of the basis state ϕ,

$$\hat{W} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

which is often called the Walsh transform, and

$$\hat{G} = -\hat{W}\hat{I}_{\bar{0}}\hat{W}$$

which effects the inversion about average. Now to perform the search on a quantum database of size N, begin with the system in the $\left|\bar{0}\right\rangle$ state and apply the $\hat{W}$ operator. This initializes all the possible states to have the same amplitude. Finally, apply the operator $\hat{G}\hat{I}_{\tau}$ (recall that operators are applied right to left), where τ is the state being sought, $\frac{\pi}{4}\sqrt{N}$ times and observe the system.

Combining the algorithms

A quantum associative memory can now be implemented by combining the two algorithms just discussed. Define $\hat{P}$ as an operator that implements the algorithm for memorizing patterns. Then the operation of the memory can be described as follows. Memorizing a set of patterns is simply

$$|\psi\rangle = \hat{P}|\bar{0}\rangle$$

with $|\psi\rangle$ being a quantum superposition of appropriate basis states, one for each pattern. Now, suppose we know *n-k* bits of a pattern and wish to recall the entire pattern. We can use a modification of Grover's algorithm to complete the pattern, producing one of the stored patterns that matches on the *n-k* bits that we know. Thus, with $2n+1$ neurons (qubits) the quantum associative memory can store up to 2^n patterns in O(*mn*) time and requires O($\sqrt{2^n}$) time to recall a pattern. This last bound is somewhat slower than desirable and may be improved with a non-unitary recall mechanism. In fact, Grover's search algorithm has been proven to be optimal in the number of steps required when unitarity is required. Thus, we have another motivation for non-unitary processes in quantum neural computation.

Recall – association

Of course, in general, a quantum memory should not only be able to complete patterns but also to correct them. In other words, given a noisy stimulus, the memory should produce the pattern most similar to that input. This can be accomplished with further modification of the basic quantum memory model we have been discussing. This modification involves the use of distributed queries and is presented in detail in [37]. Briefly, a distributed query is a distribution of the form

$$|b^p\rangle = \sum_{x=0}^{2^d-1} b_x^p |x\rangle$$

over the amplitudes of all possible states in the memory . The index *p* marks one of these states, $|p\rangle$, which is the center of the distribution (real-valued amplitudes are distributed such that the maximal value occurs at this center, and the amplitudes of the other basis states decrease monotonically with Hamming distance from the center state). This leads to the introduction of spurious memories into the recall process; however, counter to intuition the presence of these spurious memories may actually facilitate memory recall [37]. Table 3 summarizes the analogies used in developing a quantum associative memory.

Table 3. Corresponding concepts from the domains of classical neural networks and quantum associative memory

Classical neural networks		Quantum associative memory	
Neuronal State	$x_i \in \{0,1\}$	Qubit	$\lvert x\rangle = a\lvert 0\rangle + b\lvert 1\rangle$
Connections	$\{w_{ij}\}_{ij=1}^{p-1}$	Entanglement	$\lvert x_0 x_1 \ldots x_{p-1}\rangle$

Learning rule	$\sum_{s=1}^{p} x_i^s x_j^s$	Superposition of entangled states	$\sum_{s=1}^{p} a_s \lvert x_0^s \ldots x_{p-1}^s \rangle$
Winner search	$n = \max_i \arg(f_i)$	Unitary transformation	$U: \psi \to \psi'$
Output result	n	Decoherence	$\sum_{s=1} a_s \lvert x^s \rangle \Rightarrow \lvert x^k \rangle$

It should be noted that the "neuron" in the first row of the Table 3 is strictly artificial and should not be considered as a model of its biological analog. Really, as stated by Penrose "...it is hard to see how one could usefully consider a quantum superposition consisting of one neuron firing, and simultaneously not firing" [2]. There are many other arguments against attributing any biological meaning to this scheme, so we should consider it only in the context of the development of *artificial* quantum associative memory.

Implementation of QNN

How can quantum neural networks be implemented as real physical devices? First, let us mention briefly some of the difficulties we might face in the development of a physical realization of quantum neural networks.

Coherence. One of the most difficult problems in the development of any quantum computational system is the maintainence the system's coherence until the computation is complete [38]. This loss of coherence (decoherence) is due to the interaction of the quantum system with its environment. In quantum cryptography this problem may be resolved using error-correcting codes [38]. What about quantum neural networks? It has been suggested that if fact these systems may be implemented before ordinary quantum computers will be realized because of significantly lower demands on the number of qubits necessary to represent network nodes and also because of the relatively low number of state transformations required during data processing in order to perform useful computation [35, 39]. Another approach to the problem of decoherence in quantum parallel distributed processing proposed by Chrisley excludes the use of superpositional states at all and suggests the use of quantum systems for implementing standard neural paradigms, i.e. multilayer neural systems trained with backpropagation learning [20]. This model, however, takes no advantage of the use of quantum parallelism. A more promising approach to the implementation of quantum associative memory based on the use of Grover's algorithm is provided by bulk spin resonance computation (see below).

Connections. The high density of interconnections between processing elements is a major difficulty in the implementation of small-scale integration of computational systems. In ordinary neurocomputers these connections are made via wires. In (non-superpositional) quantum neurocomputers they are made via forces. In the quantum associative memory model discussed here, these connections are due to the entanglement of qubits.

Physical systems. Now we can outline what kind of physical systems might be used to develop real quantum neural networks and how these systems address the problems listed above.

- **Nuclear Magnetic Resonance.** A promising approach to the implementation of quantum associative memory based on the use of Grover's algorithm is provided by *bulk spin resonance computation*. This technique can be performed using Nuclear Magnetic Resonance systems for which coherence times on the order of thousands of seconds have been observed. Experimental verification of such an implementation has been done by Gershenfeld and Chuang [40] (among others), who used NMR techniques and a solution of chloroform ($CHCl_3$) molecules for the implementation of Grover's search on a system consisting of two qubits – the first qubit is decribed by the spin of the nucleus of the isotope C^{13}, while second one is described by the spin of the proton (hydrogen nucleus). Rather interestingly, this approach to quantum computation utilizes not a single quantum system but rather the statistical average of many copies of such a system (a collection of molecules). It is precisely for this reason that the maintenance of system coherence times is considerably greater than for true quantum implementations. Further, this technology is relatively mature, and in fact coherent computation on seven qubits using NMR has recently been demonstrated by Knill, et al. [41]. This technology is most promising in the short term, and good progress in this direction is possible in the early 21st century.
- **Quantum dots.** These quantum systems basically consist of a single electron trapped inside a cage of atoms. These electrons can be influenced by short laser pulses. Limitations to these systems which must be overcome include 1) short decoherence times due to the fact that the existence of the electron in its excited state lasts about a microsecond, and the required duration of a laser pulse is around a nanosecond; 2) the necessity of developing a technology to build computers from quantum dots of very small scale (10 atoms across); 3) the necessity of developing special lasers capable of selectively influencing different groups of quantum dots with different wavelengths of light. The use of quantum dots as the basis for the implementation of QNN is being investigated by Behrman and co-workers [16-17].
- **Other systems.** There are many other physical systems which are now being considered as possible candidates for the implementation of quantum computers (and therefore possibly quantum neurocomputers). These include various schemes of *cavity QED* (quantum electrodynamics of atoms in optical cavities), *ion traps*, *SQUID*s (superconducting quantum interference devices),

etc. Each has its own advantages and shortcomings with regard to decoherence times, speed, possibility of miniaturization, etc. More information about these technologies can be found in [4, 31].

Can QNN outperform classical neural networks?

It is now known that quantum computing gives us unprecedented possibilities in solving problems beyond the abilities of classical computers. For example Shor's algorithm gives a polynomial solution (on a quantum computer) for the problem of prime factorization, which is believed to be classically intractable [42]. Also, as previously mentioned, Grover's algorithm provides super-classical performance in searching an unsorted database.

What of quantum neural networks? Will they give us some advantages unattainable by either traditional von Neumann computation or classical artificial neural networks? Compared to the latter, quantum neural networks will probably have the following advantages:

- exponential memory capacity [30];
- higher performance for lower number of hidden neurons [39];
- faster learning [32];
- elimination of catastrophic forgetting due to the absence of pattern interference [32];
- single layer network solution of linearly inseparable problems [32];
- absence of wires [17];
- processing speed (10^{10} bits/s) [17];
- small scale (10^{11} neurons/mm^3) [17];
- higher stability and reliability [39];

These potential advantages of quantum neural networks are indeed compelling motivation for their development. However, the more remote future possibilities of QNN may be even more exciting.

Frontiers of QNN

It is generally believed that the right hemisphere is responsible for spatial orientation, intuition, semantics etc., while the left hemisphere is responsible for temporal processing, logical thinking and syntax. Given this view, it is very natural to consider that neurocomputers can be thought of as imitating our right brain function while von Neumann computers can be thought as as mimicing the functionality of our left brain. Penrose characterizes these two types of computation as *bottom-up* and *top-down* respectively. Nevertheless, he argues that higher brain functions such as consciousness cannot be modelled using just these types of computation. The ideas discussed in this chapter introduce the possibility of combining the unique computational abilities of classical neural networks and quantum computation, thus producing a computational paradigm of incredible potential. However, we make no effort here to relate any of these concepts to biological systems; in fact, much of what we have discussed is most likely very different from biological neural information processing. Therefore it seems unlikely that quantum neural networks, at least in the context discussed here, could be considered a candidate for the basis of consciousness. However, Perus has suggested that neural networks can be a "macroscopic replica of quantum processing structures". If so, they "could be an interface between the macro-world of man's environment and the micro-world of his non-local consciousness" [43]. Thus, it is not out of the realm of possibility that future models of quantum neural networks may afterall provide significant insight into the workings of the mind and brain.

There are some proponents for the idea that QNN may be developed that have abilities beyond the restrictions imposed by the Church-Turing thesis. Simply put, according to this thesis, all existing computers are equivalent in computational power to the Universal Turing Machine. Moreover, all *algorithmic* processes we can perform in our mind can be realized on this machine and *vice versa*. No existing neurocomputers, nor any quantum computers theorized to date can escape the bounds imposed by the Church-Turing thesis. But what about quantum neural networks? Dan Cutting has posed the query, "*Would quantum neural networks be subject to the decidability constraints of the Church-Turing thesis*?" [39]. For existing models of QNN the answer seems surely to be "no", but some speculative physical systems (wormholes, for example) are discussed as possible candidates for the basis of QNN that could exceed these bounds [39]. This is a very intriguing question, and it is a challenge for the future to try to develop a theory of quantum neural networks that will give us completely new computational abilities for tackling problems that cannot now be solved even in principle. In the process we shall certainly be examining the concept of computation in a very different light and in so doing will be likely to make discoveries that to this point have been overlooked.

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